UTILIZATION OF STRUCTURAL PARAMETERS TO INVESTIGATE THE THERMOSTRESS STATE OF A DEFORMABLE BODY UNDER PULSE HEATING

V. S. Zarubin and G. N. Kuvyrkin

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Equations to determine the temperature and stress in a solid body under nonstationary loading are obtained from the relations of the thermodynamics of irreversible processes with internal structural parameters.

The intensification of thermal effects on structure elements and the creation of new technological processes based on utilization of high-intensity pulsed energy flows stimulate the creation of thermomechanical models of the behavior of the materials being used. Depending on the loading and exploitation conditions, and the assurance of the needed accuracy of the computations, these models should take account of the effects of viscoelasticity, plasticity, and creep, cumulative damage, finiteness of the thermal propagation rate, etc., under condition of high variable temperatures. Three fundamental approaches, based on examination of media of velocity type, media with memory, and media with internal structural parameters [1], are utilized at this time to obtain the governing equations describing the behavior of materials under such conditions. The most general here is the approach based on utilization of models of media with memory whose principal disadvantage is the mathematical formalism which often masks the physical crux of the phenomena under consideration. The application of models of media with internal structural parameters has a number of advantages associated primarily with the possibility of describing the behavior of macroscopic objects with microstructural processes proceeding at the molecular and submolecular levels [1, 2], taken into account.

The structural parameters can be both scalar and vector and tensor quantities. Thus, for crystalline materials the statistically averaged tensor quantities of dislocation and internal stress densities and the scalar quantity of cumulative inelastic strain [2-4] can be taken as the structural parameters. For materials with high-molecular structure the application of external loads causes uncoiling and reorientation of the molecular chains, and redistribution of the number of molecular segments occurs between the ordered and disordered parts of a polymer [5]. At the macrolevel this is manifest in the form of quite definite viscous properties of polymers. Under conditions of highly intensive thermomechanical action, disorder of the structure, disturbance of the bounds between the structural elements and the formation of microcracks, micropores, etc., occur in the material, i.e., cumulative damage. In describing the process of heat conduction on the basis of representations about phonon gas motion, the structural parameter can be associated with the vector phonon distribution function subjected to a kinetic equation of relaxation type [6]. In the general case, if the characteristic time of the change in the external load is close in magnitude to the relaxation time of the structural parameter, taking account of the change in the internal structural parameters is necessary [7].

Let us assume that the thermostress state of a deformable body is determined by four thermodynamic functions [1, 4]: the free energy $A = A(\varepsilon_{kl}, T, \chi^{(S)}, \chi_k^{(T)}, \chi_{kl}^{(\sigma)}, \vartheta_k)$, the entrop $S = S(\varepsilon_{kl}, T, \chi^{(S)}, \chi_k^{(T)}, \chi_{kl}^{(\sigma)}, \vartheta_k)$, the stress tensor, and the thermal flux density vector, respectively, with the components $\sigma_{ij} = \sigma_{ij}(\varepsilon_{kl}, T, \chi^{(S)}, \chi_k^{(T)}, \chi_{kl}^{(\sigma)}, \vartheta_k)$ and $q_i = q_i(\varepsilon_{kl}, T, \chi^{(S)}, \chi_k^{(T)}, \chi_{kl}^{(\sigma)}, \vartheta_k)$.

The existence of the kinetic equations

$$\dot{\chi}^{(S)} = \varkappa^{(S)}(\varepsilon_{kl},\ T,\ \chi^{(S)},\ \chi_k^{(T)},\ \chi_{kl}^{(\sigma)},\ \vartheta_k), \tag{1}$$

$$\dot{\chi}_i^{(T)} = \varkappa_i^{(T)}(\varepsilon_{kl},\ T,\ \chi^{(S)},\ \chi_k^{(T)},\ \chi_{kl}^{(\sigma)},\ \vartheta_k), \tag{2}$$

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$$\dot{\chi}_{ij}^{(\sigma)} = \varkappa_{ij}^{(\sigma)}(\varepsilon_{kl},\ T,\ \chi^{(S)},\ \chi_k^{(T)},\ \chi_{kl}^{(\sigma)},\ \vartheta_h) \quad (3)$$

is postulated to determine the structural parameters. Moreover, at each point of the body there should be satisfied [1, 2, 4]

The Conservation of Momentum Law

$$\rho\ddot{u}_i - \sigma_{ji,j} = F_i; \quad (4)$$

The Moment of Momentum Conservation Law

$$\sigma_{ij} = \sigma_{ji}; \quad (5)$$

The Energy Conservation Law (The First Law of Thermodynamics)

$$\rho\dot{U} + q_{i,i} - \sigma_{ij}\dot{\varepsilon}_{ij} - \rho r = 0; \quad (6)$$

The Second Law of Thermodynamics (The Clausius-Duhem Inequality)

$$\rho T\dot{S} + T(q_i/T)_{,i} - \rho r \geqslant 0. \quad (7)$$

In the case of small deformation the continuity equation is satisfied automatically.

If the connection between the internal and free energies U = A + TS is utilized, then taking account of the inequality (7), there follows from (6) by virtue of the arbitrariness of $\dot{\varepsilon}_{ij}$, $\dot{T}$ and $\dot{\vartheta}_i$

$$\sigma_{ij} = \rho\frac{\partial A}{\partial\varepsilon_{ij}},\quad S = -\frac{\partial A}{\partial T},\quad \frac{\partial A}{\partial\vartheta_i} = 0 \quad (8)$$

and the second law of thermodynamics acquires the form

$$\rho\frac{\partial A}{\partial\chi^{(S)}}\dot{\chi}^{(S)} + \rho\frac{\partial A}{\partial\chi_i^{(T)}}\dot{\chi}_i^{(T)} + \rho\frac{\partial A}{\partial\chi_{ij}^{(\sigma)}}\dot{\chi}_{ij}^{(\sigma)} + q_iT_{,i}/T \leqslant 0, \quad (9)$$

i.e., the kinetic equations (1)-(3) cannot be arbitrary, their specific form should be selected with the inequality (9) taken into account.

Taking account of (8) the energy conservation law (6) is written in the form

$$\rho T\dot{S} + \rho\frac{\partial A}{\partial\chi^{(S)}}\dot{\chi}^{(S)} + \rho\frac{\partial A}{\partial\chi_i^{(T)}}\dot{\chi}_i^{(T)} + \rho\frac{\partial A}{\partial\chi_{ij}^{(\sigma)}}\dot{\chi}_{ij}^{(\sigma)} + q_{i,i} - \rho r = 0. \quad (10)$$

Henceforth, not only the total deformations ($\varepsilon_{ij} \ll 1$), but also the temperature deformations and the structural parameters are assumed small $\varepsilon_{ij}^{(T)} \ll 1$, $\chi^{(S)} \ll 1$, $\chi_i^{(T)} \ll 1$ and $\chi_{ij}^{(\sigma)} \ll 1$. Then the free energy can be written in a form analogous to that proposed in [8] for a medium with memory

$$A(\varepsilon_{hl},\ T,\ \chi^{(S)},\ \chi_k^{(T)},\ \chi_{kl}^{(\sigma)}) = A^*(\varepsilon_{kl} - \varepsilon_{kl}^{(T)},\ \chi^{(S)},\ \chi_k^{(T)},\ \chi_{kl}^{(\sigma)}) + B(T) - A^*(-\varepsilon_{kl}^{(T)}). \quad (11)$$

Furthermore, representing the first and third components in the right side of (11) in the form of a Taylor series in the corresponding arguments and limiting ourselves to quadratic terms in the expansions, we can write

$$\begin{aligned}
\rho A(\varepsilon_{kl},\ T,\ \chi^{(S)},\ \chi_k^{(T)},\ \chi_{kl}^{(\sigma)}) = {} & \frac{1}{2}C_{ijkl}(\varepsilon_{kl} - \varepsilon_{kl}^{(T)})(\varepsilon_{ij} - \varepsilon_{ij}^{(T)}) + \\
& + \frac{1}{2}F(\chi^{(S)})^2 + \frac{1}{2}K_{ij}\chi_j^{(T)}\chi_i^{(T)} + \frac{1}{2}H_{ijkl}\chi_{kl}^{(\sigma)}\chi_{ij}^{(\sigma)} - \\
& - G_{ij}\chi^{(S)}(\varepsilon_{ij} - \varepsilon_{ij}^{(T)}) - N_{ijk}\chi_k^{(T)}(\varepsilon_{ij} - \varepsilon_{ij}^{(T)}) - \\
& - M_{ijkl}\chi_{kl}^{(\sigma)}(\varepsilon_{ij} - \varepsilon_{ij}^{(T)}) - P_i\chi^{(S)}\chi_i^{(T)} - Q_{ij}\chi^{(S)}\chi_{ij}^{(\sigma)} - \\
& - L_{ijk}\chi_k^{(T)}\chi_{ij}^{(\sigma)} + \rho B(T) - \frac{1}{2}C_{ijkl}\varepsilon_{kl}^{(T)}\varepsilon_{ij}^{(T)},\quad A(0,\ T_0,\ 0,\ 0,\ 0) = 0.
\end{aligned} \quad (12)$$

Evidently the coefficients for the linear terms in (12) should be zero. The expressions for the stress tensor and the entropy will take the following form when the first two equalities from (8) are taken into account

$$\sigma_{ij} = C_{ijkl}(\varepsilon_{kl} - \varepsilon_{kl}^{(T)}) - G_{ij}\chi^{(S)} - N_{ijk}\chi_k^{(T)} - M_{ijkl}\chi_{kl}^{(\sigma)}, \tag{13}$$

$$S = \frac{1}{\rho}(\sigma_{ij} + C_{ijkl}\varepsilon_{kl}^{(T)})\frac{\partial\varepsilon_{ij}^{(T)}}{\partial T} - \frac{\partial B}{\partial T}, \tag{14}$$

and taking (10) and (14) into account, another form of writing the energy conservation law can be obtained

$$\begin{aligned} \dot{T}T\frac{\partial}{\partial T}\left[(\sigma_{ij} + C_{ijkl}\varepsilon_{kl}^{(T)})\frac{\partial\varepsilon_{ij}^{(T)}}{\partial T}\right] + (C_{ijkl}\dot{\varepsilon}_{kl} - G_{ij}\dot{\chi}^{(S)} - \\ - N_{ijk}\dot{\chi}_k^{(T)} - M_{ijkl}\dot{\chi}_{kl}^{(\sigma)})\,T\frac{\partial\varepsilon_{ij}^{(T)}}{\partial T} - \rho T\frac{\partial^2 B}{\partial T^2}\dot{T} + \\ + \rho\frac{\partial A}{\partial\chi^{(S)}}\dot{\chi}^{(S)} + \rho\frac{\partial A}{\partial\chi_i^{(T)}}\dot{\chi}_i^{(T)} + \rho\frac{\partial A}{\partial\chi_{ij}^{(\sigma)}}\dot{\chi}_{ij}^{(\sigma)} + q_{i,i} - \rho r = 0. \end{aligned} \tag{15}$$

All the coefficients in (12)-(15) are taken temperature-dependent.

To obtain the energy conservation law in the form of the heat conduction equation it is necessary to make specific the expressions for the heat flux density vector and the structural parameters by taking them, say, in the form

$$q_i = \varphi_i^{(S)}\chi^{(S)} + \varphi_{ij}^{(T)}\chi_j^{(T)}; \tag{16}$$

$$\tau_S\dot{\chi}^{(S)} = -\chi^{(S)} + \bar{\chi}^{(S)} \quad \text{or} \quad \chi^{(S)} = \bar{\chi}^{(S)} - \int_0^t \exp\left(-\frac{t-t'}{\tau_S}\right)\frac{\partial\bar{\chi}^{(S)}}{\partial t'}dt'; \tag{17}$$

$$\tau_T\dot{\chi}_i^{(T)} = -\chi_i^{(T)} + \bar{\chi}_i^{(T)} \quad \text{or} \quad \chi_i^{(T)} = \bar{\chi}_i^{(T)} - \int_0^t \exp\left(-\frac{t-t'}{\tau_T}\right)\frac{\partial\bar{\chi}_i^{(T)}}{\partial t'}dt'; \tag{18}$$

$$\tau_\sigma\dot{\chi}_{ij}^{(\sigma)} = -\chi_{ij}^{(\sigma)} + \bar{\chi}_{ij}^{(\sigma)} \quad \text{or} \quad \chi_{ij}^{(\sigma)} = \bar{\chi}_{ij}^{(\sigma)} - \int_0^{\bar{t}} \exp\left(-\frac{t-t'}{\tau_\sigma}\right)\frac{\partial\bar{\chi}_{ij}^{(\sigma)}}{\partial t'}dt'. \tag{19}$$

If it is assumed that a change in the structural parameter occurs at the constant temperature T_*, then analogously to what is proposed in [9]

$$\bar{\chi}^{(S)} = H(T - T_*), \tag{20}$$

which corresponds to a phase transition of the first kind occurring at constant temperature. In this case the components of the heat flux density vector (16) undergo discontinuity, equal numerically to $\varphi_i^{(S)}$ and corresponding to the heat of the phase transition (as $\tau_S \to 0$). The stress tensor components on the surface of the phase transition also undergo discontinuity whose magnitude will equal G_{ij}, as will follow from (13). If $\bar{\chi}^{(S)}$ is assumed dependent on ε_{ij}, T, and ϑ_i, then the kinetic equations (17) are analogous to those examined in [10]. In the general case, the process of a change in the structural parameter $\chi^{(S)}$ can be treated as a cumulative damage process in a material and the parameter itself as the damage [11], where $0 \leq \chi^{(S)} \leq 1$.

Since the phonon vector distribution function in the steady state is proportional to the temperature gradient [6]

$$\bar{\chi}_i^{(T)} = Z_{ij}\vartheta_j, \tag{21}$$

then in the absence of structural changes ($\chi^{(S)} = 0$) the equation (16), with the second relationship from (18) and the equality (21) taken into account, takes the form

$$q_i = -\lambda_{ij}^{(T)}\vartheta_j + \int_0^t \exp\left(-\frac{t-t'}{\tau_T}\right)\frac{\partial}{\partial t'}(\lambda_{ij}^{(T)}\vartheta_j)\,dt', \tag{22}$$

corresponding to the heat propagation process at a finite velocity. The dependence (22) was obtained in [12] by starting from other considerations. Substituting (22) into (15) and neglecting the coherence effects by virtue of their smallness [13], and the negligible influence on the temperature distribution, we write the heat conduction equation in the form

$$\rho c_\varepsilon \dot{T} = (\lambda_{ij}^{(T)} T_{,j})_{,i} - \int_0^t \exp\left(-\frac{t-t'}{\tau_T}\right) \frac{\partial}{\partial t'} (\lambda_{ij}^{(T)} T_{,j})_{,i} dt' + \rho r. \tag{23}$$

As a rule, the relaxation time τ_T is a sufficiently small quantity, and a method developed in [14, 15] can be used for its determination.

Under highly intensive loading the change in the structural parameters $\chi_{ij}^{(\sigma)}$ characterizes the internal viscosity of the medium being examined. Then in the absence of structural changes ($\chi^{(S)} = 0$) and neglecting the influence of the heat propagation process on the stress-strain state ($N_{ijk} = 0$) for

$$\bar{\chi}_{ij}^{(\sigma)} = X_{ijkl} \varepsilon_{kl} \tag{24}$$

there follows from the equality (13)

$$\sigma_{ij} = C_{ijkl}(\varepsilon_{kl} - \varepsilon_{kl}^{(T)}) - R_{ijkl}\varepsilon_{kl} + \int_0^t \exp\left(-\frac{t-t'}{\tau_\sigma}\right) \frac{\partial}{\partial t'} (R_{ijkl}\varepsilon_{kl})\, dt'. \tag{25}$$

The dependence (25) describes a standard linear medium [16], but with the temperature deformation taken into account. If the influence of the structural parameter $\chi_{ij}^{(\sigma)}$ on the stress is not taken into account, i.e., $R_{ijkl} = 0$, then (25) describes a linearly elastic medium.

It is expedient to analyze the role of each structural parameter separately in computations of the temperature and stress-strain states of a body under investigation.

The following problem

$$\dot{\theta} = \partial^2\theta/\partial z^2, \qquad \bar{t} = 0 \quad \theta(z,\ 0) = 0; \tag{26}$$

$$z = 0 \quad -\partial\theta/\partial z = q_0(\bar{t}),\ \theta(0,\ \bar{t}) < \theta_*,\ q_0(\bar{t}) = M\bar{t}^m \exp(-m\bar{t}); \qquad z \to \infty \quad \theta(z,\ \bar{t}) \to 0 \tag{27}$$

is considered to estimate the influence of the relaxation time relationship of the structural parameter $\chi^{(S)}$ and the position of the maximum of the time-variable deliverable thermal flux pulse on the magnitude of the mass entrainment from the surface of the body being heated.

If $\theta(0, \bar{t}) = \theta_*$ ($\bar{t} \geq \bar{t}_*$), then the boundary condition on the heated surface will be

$$z = h(\bar{t}) \quad -\partial\theta/\partial z = q_0(\bar{t}) - \dot{h}Q_*\chi^{(S)}, \qquad \chi^{(S)}(\bar{t}) = \left[1 - \exp\left(-\frac{\bar{t} - \bar{t}_*}{D_S^2}\right)\right] H(\theta - \theta_*). \tag{28}$$

The selection of the exponent for the exponential and the coefficient M in the expression for $q_0(\bar{t})$ assures the maximum of this function for $\bar{t} = 1$ for all m and moreover $\int_0^\infty q_0(\bar{t})\, d\bar{t} = 1$.

The solution of the boundary value problem (26)-(28) can be obtained by the integral heat balance method [17], where in the first heating stage

$$\theta(z,\ \bar{t}) = \frac{1}{2} q_0(\bar{t})\, \delta_1(\bar{t}) [1 - z/\delta_1(\bar{t})]^2,\ \delta_1(\bar{t}) = \sqrt{6 \int_0^{\bar{t}} q_0(u)\, du / q_0(\bar{t})}, \tag{29}$$

and in the second

$$\theta(z,\bar{t})=\theta_*[1-(z-h)/(\delta_2-h)]^2,\quad h(\bar{t})=\int_0^{\bar{t}}\Big[-\frac{2\theta_*}{\sqrt{\delta_1^2(\bar{t}_*)+12\theta_*(u-\bar{t}_*)}}+ \\ +q_0(u)\Big]\frac{du}{Q_*\chi^{(S)}(u)},\quad \delta_2(\bar{t})=h(\bar{t})+\sqrt{\delta_1^2(\bar{t}_*)+12\theta_*(\bar{t}-\bar{t}_*)}. \tag{30}$$

Calculation of the magnitude of the surface heat entrainment $h(\bar{t})$ is performed until the integrand is non-negative, i.e., $\bar{t} \le \bar{t}_{**}$.

Represented in Fig. 1 are results of computations of $h(\bar{t}_{**})$ as a function of D_S^2 for a number of values of Q_* for a fixed phase transition temperature θ_*. As the parameter D_S^2 grows, starting with $D_S^2 = 0.1$, the thickness of the entrained layer of material evidently increases abruptly, where all the more, the smaller the phase transition heat. An increase in θ_* results, respectively, in a reduction in $h(\bar{t}_{**})$ without altering the nature of the dependence on D_S^2.

The influence of the relationship of the relaxation time τ_T of the structural parameter $\chi_i^{(T)}$ and the position of the maximum of the deliverable heat flux $q_0(\bar{t})$ on the temperature distribution and stress in an elastic isotropic body can be investigated by solving the heat conduction and motion equations

$$D_T^2\ddot{\theta}+\dot{\theta}=\partial^2\theta/\partial z^2, \tag{31}$$

$$R^2(\ddot{\sigma}+\ddot{\theta})=\partial^2\sigma/\partial z^2 \tag{32}$$

with the boundary conditions

$$\begin{aligned}&\bar{t}=0\quad \theta(z,0)=0,\ \dot{\theta}(z,0)=0,\ \sigma(z,0)=0,\ \dot{\sigma}(z,0)=0;\\ &z=0\quad -\partial\theta/\partial z=q_0(\bar{t})+D_T^2\dot{q}_0(\bar{t}),\ \sigma(0,\bar{t})=0;\\ &z\to\infty\quad \theta(z,\bar{t})\to 0,\ \sigma(z,\bar{t})\to 0.\end{aligned} \tag{33}$$

The solutions of (31) and (32) with the boundary conditions (33) have the form

$$\theta(z,\bar{t})=\int_0^{\bar{t}}\Big[D_T\dot{q}_0(\bar{t}-u)+\frac{1}{D_T}q_0(\bar{t}-u)\Big]F_1(z,u)\,du; \tag{34}$$

$$\sigma(z,\bar{t})=\frac{R^2}{D_T^2-R^2}\int_0^{\bar{t}}\Big[D_T\dot{q}_0(\bar{t}-u)-\frac{R^2}{D_T(D_T^2-R^2)}q_0(\bar{t}-u)+ \\ +\frac{R^2}{D_T(D_T^2-R^2)^2}\exp\Big(-\frac{\bar{t}-u}{D_T^2-R^2}\Big)\int_0^{\bar{t}-u}q_0(v)\exp\Big(\frac{v}{D_T^2-R^2}\Big)dv\Big]\times[F_1(z,u)-F_2(z,u)]\,du; \tag{35}$$

$$F_1(z,u)=\exp\Big(-\frac{u}{2D_T^2}\Big)I_0\Big(\frac{\sqrt{u^2-z^2D_T^2}}{2D_T^2}\Big),$$

$$F_2(z,u)=\exp\Big(-\frac{u-zR}{2D_T^2}\Big)I_0\Big(\frac{u-zR}{2D_T^2}\Big),$$

$F_1(z,u)\ne 0$ only for $u > zD_T$ and $F_2(z,u)\ne 0$ only for $u > zR$.

If the speed of sound V_σ agrees with the rate of heat propagation, i.e., $R^2/D_T^2 = 1$, or the rate of heat propagation is infinite ($D_T^2 = 0$), then the solution of (31) and (32) are easily obtained by an analogous means by using the Laplace transform.

Represented in Fig. 2 are results of temperature computations for the time $\bar{t} = 5$ when practically all the energy supplied is absorbed by the body. The degree of heat flux penetration here is $z = \bar{t}/D_T$. As D_T^2 diminishes the solution of the hyperbolic heat conduction equation approximates the solution of the parabolic equation ($D_T^2 = 0$). Computations show that $D_T^2 = 0$ can be assumed for $D_T^2 < 0.001$ for the solution of heat conduction equations for any

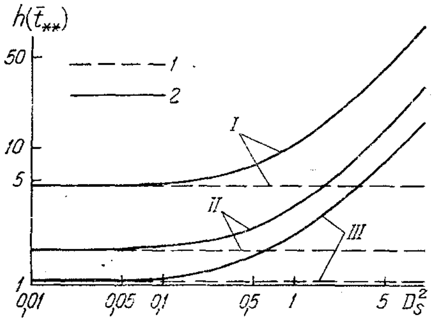

Fig. 1

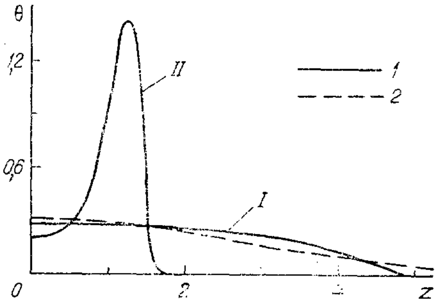

Fig. 2

Fig. 1. Dependence of the magnitude of the surface mass entrainment $h(\bar{t}_{**})$ on the parameter D_S^2 for $\theta_* = 0.1$ and m = 2 for a number of values of Q_*: 1) Without taking account of the influence of the change in the structural parameters $\chi^{(S)}$; 2) with the influence of this change taken into account; I, II, III are $Q_* = 0.1$, 0.3, and 0.5, respectively.

Fig. 2. Temperature distribution in the body for m = 2 for $\bar{t} = 5$: 1) From (34); 2) parabolic temperature distribution ($D_T^2 = 0$); I, II are $D^2 = 1$, 10, respectively.

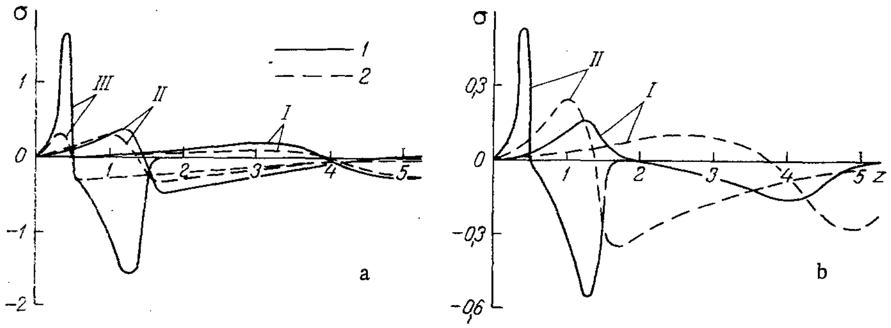

Fig. 3. Stress distribution in an elastic body for $R^2/D_T^2 = 10$ (a) and $R^2/D_T^2 = 0.1$ (b), m = 2 for $\bar{t} = 5$: 1) from (35); 2) corresponding to a parabolic temperature distribution ($D_T^2 = 0$): a) I, II, III are $R^2 = 1$, 10, 100; b) I, II are $R^2 = 1$, 10.

time $\bar{t}$. Represented in Fig. 3a are results of stress computations at that same time for $R^2/D_T^2 = 10$ (the sound speed is less than the heat propagation rate) and the same values of the parameter D_T^2, as in Fig. 2. It is seen from the results represented that the positions of the extremums of the appropriate stresses are nearby, however the difference in their magnitude grows substantially as the parameter R^2 increases. A sharp change in the derivative $\partial\sigma/\partial z$ corresponds to a point with the coordinate $z = \bar{t}/R$. If the sound speed is greater than the heat propagation rate ($R^2/D_T^2 = 0.1$) then the positions of the stress extremums are far apart for identical values of R^2 (Fig. 3b), while the stresses for $D_T^2 = 0$ and $D_T^2 \neq 0$ do not differ so radically. Moreover, if the sound speed is less than the heat propagation rate ($R^2/D_T^2 > 1$), then the absolute values of the stresses exceed the corresponding values for $R^2/D_T^2 < 1$ significantly.

Taking account of the material viscosity (the structural parameter $\chi_{ij}^{(\sigma)}$) results in a certain reduction in the stress values without changing either the positions of the extremums or the points of an abrupt change in the derivatives $\partial\sigma/\partial z$. To obtain a qualitative estimate of the influence of the structural parameter $\chi_{ij}^{(\sigma)}$ on the stress, the homogeneous stress state can be considered when the deformation changes according to the law

$$\varepsilon(\bar{t}) = M\bar{t}^m \exp(-m\bar{t}). \tag{36}$$

In this case the stress will be

$$\sigma(\bar{t}) = \varepsilon(\bar{t}) - \beta\left[\varepsilon(\bar{t}) - \int_0^{\bar{t}} \exp\left(-\frac{\bar{t}-u}{D_\sigma^2}\right)\frac{\partial\varepsilon}{\partial u}\,du\right]. \tag{37}$$

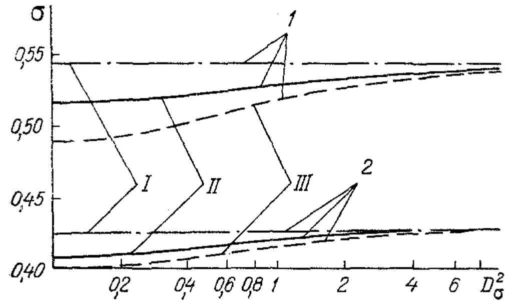

Fig. 4. Dependence of the stress on D_σ^2 for the times $\bar{t} = 1$ (1) and $\bar{t} = 2 - \sqrt{2}$ (2) for m - 2; I, II, III — β = 0; 0.05 and 0.1.

Represented in Fig. 4 are certain results of computing $\sigma(\bar{t})$ as a function of D_σ^2 for two times corresponding to the maximum deformation ($\bar{t} = 1$) and the maximal value of the rate of a train change ($\bar{t} = 2 - \sqrt{2}$). For $\beta = 0$ the material becomes linearly elastic. If $D_\sigma^2 \to \infty$, then the first factor under the ingegral in (37) tends to one and $\sigma(\bar{t}) \to \varepsilon(\bar{t})$. For $D_\sigma^2 \to 0$ the stress tends to the steady state value $\sigma(\bar{t}) = (1 - \beta)\varepsilon(\bar{t})$. In those cases when D_σ^2 is commensurate with the magnitude of the time interval under consideration, taking account of the relaxation effects evidently is necessary for pulse loading. As follows from Fig. 4, these effects appear most clearly for large values of β.

NOTATION

$A = A(\varepsilon_{hl}, T, \chi^{(S)}, \chi_k^{(T)}, \chi_{kl}^{(\sigma)}, \vartheta_k)$ is the free energy; $S = S(\varepsilon_{hl}, T, \chi^{(S)}, \chi_k^{(T)}, \chi_{kl}^{(\sigma)}, \vartheta_k)$ is the entropy; $\sigma_{ij} = \sigma_{ij}(\varepsilon_{hl}, T, \chi^{(S)}, \chi_k^{(T)}, \chi_{kl}^{(\sigma)}, \vartheta_h)$ are the stress tensor components; $q_i = q_i(\varepsilon_{hl}, T, \chi^{(S)}, \chi_k^{(T)}, \chi_{kl}^{(\sigma)}, \vartheta_h)$ are the components of the heat flux density vector; ε_{ij}, are small strain tensor components; T, T_0, are the temperature and initial temperature; $\chi^{(S)}$ is the structural parameter characterizing the disorder of the structure (phase transitions, cumulative damage, etc.); $\chi_k^{(T)}$, $\chi_{kl}^{(\sigma)}$ are structural parameters characterizing the heat propagation and stresses, respectively; $\vartheta_k = T_k$ is the temperature gradient; $(\ )_{,k} = \partial/\partial x_k$; x_k are Cartesian coordinates (i, j, k, l = 1, 2, 3); $(\cdot) = \partial/\partial t$; t is the time; ρ is the density; U is the internal energy per unit mass; r is the mass density of internal heat liberation; $\varepsilon_{ij}^{(T)}$ are components of the small temperature strain tensor; B(T) is the thermal part of the internal energy; τ_S, τ_T, τ_σ are the relaxation times of the structural parameters $\chi^{(S)}$, $\chi_k^{(T)}$, and $\chi_{kl}^{(\sigma)}$, respectively; $H(T - T_*)$ is the Heaviside function; T_* is the temperature of the phase transition of the first kind; $\lambda_{ij}^{(T)} = -\varphi_{ik}^{(T)} Z_{kj}$ is the heat conduction tensor; $c_\varepsilon = -T\partial^2 B/\partial T^2$ is the specific heat under constant strain; $a = \lambda^{(T)}/(\rho c_\varepsilon)$ is the thermal diffusivity; t_0 is the coordinate of the maximum of the delivered heat flux $Q_0(t)$; $V_\sigma = \sqrt{(\lambda + 2\mu)/\rho}$ is the sound speed in the material; λ, μ, are elastic constants; $M = m^m/(m - 1)!$; $T^* = At_0^m\sqrt{at_0}/\lambda^{(T)}$; $D_T^2 = \tau_T/t_0$; $q_0 = Q_0/(At_0^m)$; $\theta = (T - T_0)/T^*$; $R^2 = a/(t_0V^2)$; $\bar{t} = t/t_0$; $z = x/\sqrt{at_0}$; $\sigma = \sigma_{11}/(3\lambda + 2\mu)\alpha T^*$; α is the temperature coefficient of linear expansion; $I_0(\ )$ is the modified Bessel function $\operatorname{erf}(y) = \frac{2}{\sqrt{\pi}}\int_0^y \exp(-x^2)\,dx$ is the error function; $\operatorname{erfc}(y) = 1 - \operatorname{erf}(y)$; θ_*, Q_* are the dimensionless temperature and heat of the phase transition of the first kind; $D_S^2 = \tau_S/t_0$; $\beta = (R_1 + 2R_2)/(\lambda + 2\mu)$; $R_{ijkl} = R_1\delta_{ij}\delta_{kl} + R_2(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$; $D_\sigma^2 = \tau_\sigma/t_0$; $\sigma = \sigma_{11}/[(\lambda + 2\mu)A_1t_0^m]$ are dimensionless stresses (Fig. 4).

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THERMAL STRESSES AND DEFORMATIONS IN A PLATE SUBJECT TO THE ACTION OF CONCENTRATED ENERGY FLOWS

G. I. Rudin

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A two-dimensional problem is solved concerned with the determination of temperature and stress fields in a plate subject to heating by a radiative flow of Gaussian type.

Nonhomogeneous radiative heating of a plate induces thermal stresses and deformations in the plate. If the intensity of the radiative flow is sufficiently high, the stresses may exceed the limit of strength of the plate material, giving rise to irreversible structural changes in the plate. In particular, the role of the thermal deformations manifests itself in a twisting of the plate surface. If the plate is an element of an optical system, this effect leads to a distortion in the structure of the beam being transmitted, for example, to a lack of focus. There is also increased interest in the study of stresses and deformations under the action of concentrated flows of radiation with a Gaussian distribution of intensity along a radius when the radius of the zone of exposure is equal in order of magnitude or significantly less than the plate thickness. In this case the spatial distribution of stresses and deformations is two-dimensional and differs essentially from the one-dimensional approximation.

In [1] a two-dimensional problem was treated concerned with the determination of the stresses in a free plate under the action of a thermal surface source. At the same time, there is considerable practical interest in the study of stress and deformation fields when the thermal source is a volume source. Such sources are formed, in particular, under the action of a laser beam on a nonmetallic material, and the action of an electron-beam flow on metal. In these cases, a thermal source is formed in the plate whose strength depends expon-

A. V. Lykov Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 54, No. 3, pp. 477-483, March, 1988. Original article submitted September 29, 1986.

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